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A Linear Programming Model

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# Introduction

This is Microsoft Word Report accompanying my R Script and Excel Worksheet. In this analysis, my main aim is to utilize linear programming in order to maximize profit for a company. I was given specific business scenario and asked to devise a linear model in order to maximize profit and optimize cost. Firstly, I identified my decision variables, constraints and objective function and formulated a mathematical model. Then, I used R and LPsolveAPI package in order to implement a linear programming to optimize my mathematical model. Finally, I used Excel for sensitivity analysis for decision variables.

Since I also provided R script and Excel worksheet with all the codes and comments, I removed some of the codes and comments from my report (such as package loading). Also, I did not incorporate all of my R code to report since it was too lengthy. However, all necessary outputs (Graphs and results) are added. It is due to keep my report brief, succinct and to the point.

# Mathematical Formulation

To start with, I used given data in order to create a table which clearly shows cost, selling price and profit for each of 4 products (Table 1.1). At first, I observed that, Go Kart makes the most profit per unit. But, since there are many constraints , we cannot deduce that company should only sell Go Karts.

*Table 1.1 – Monetary Information for each Product*

|  |  |  |  |
| --- | --- | --- | --- |
| Item | Cost in USD | Price (per unit) | Profit |
| Pressure Washer | *320* | *439.99* | *119.99* |
| Go Kart | *360* | *719.99* | *359.99* |
| Generator | *415* | *639.99* | *224.99* |
| Case of 5 Water Pumps | *637* | *269.99* | *712.95* |

Our first constraint is obviously cost. According to company policy, they allocated 176000 USD/month for buying of new products. Secondly, company has a limited space in their inventory. It is limited to 84 shelves with 32ft long x 5 ft wide dimensions. Dimensions of each product is clearly specified in Table 1.2. Here, since all products has width of 5 ft ,which is exactly same with shelves’ width, I will only consider their length as a constraint since width values fully occupy shelves. In this case, our area constraint is 84 shelves with 32ft length. Third constraint is about marketing campaign which says that, company should allocate at least 27% of their inventories to Pressure Washers and Go Karts. Considering total length of 2688 ft (84 x 32 ft), it is about 725.76 ft. Finally, company must sell at least twice as many generators as water pumps (We must keep in mind that company order pumps in cases which holds 5 individual water pumps. ).

*Table 1.2 – Sizes of each Product*

|  |  |  |
| --- | --- | --- |
| Item | Length in ft | Wide |
| Pressure Washer | *3* | 5 |
| Go Kart | 8 | 5 |
| Generator | 3 | 5 |
| 4 Cases of Water Pumps | 3 | 5 |

With all these being said, I proceed to Mathematically formulate my model. In order to mathematically formulate my model, I called Pressure Washer, Go Kart, Generator and case of water pumps to be x1, x2, x3, and x4 respectively. Then my objective function, which is profit, is as follow :

(1)

Moreover, I formulated my constraints as follow :

(2)

(3)

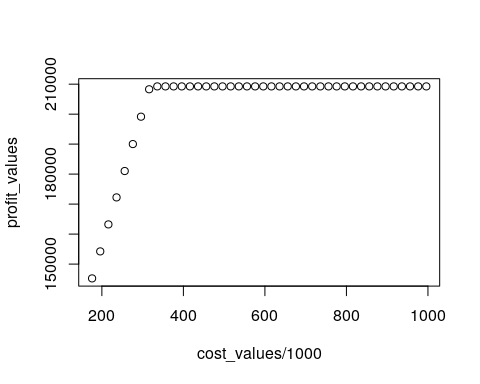
(4)

(5)

One important point is that, for Area constraint (Equation 2), I said both of x1 and x3 takes 3.2 ft instead of 3ft. The reason for that, since length of shelve is 32 ft and it can only contain 10 of x1 or x3, there are 2 ft excess area. But when considering 2 shelves, there is 4 ft empty area which , theoretically , can contain 1 more Pressure Washer or Generator. But, since we cannot separate them into 2 pieces like 2ft and 1ft, this area cannot fit any item. In order to solve this inconsistency, I assumed Pressure washer and Generator takes 3.2 ft (32 ft / 10 items). On the other hand, when it comes to Water Pumps, 4 case, which means 4 \* x4, occupy 3.2 ft area (3.2 ft because above reasoning). So one case of Water pump occupies only 0.8 ft area.

After formulating my objective function, decision variables and constraints, I utilized R and lpSolveAPI package in order to perform monthly analysis. After creating and solving my function in R, I found that optimal values for Pressure Washer, Go Kart, Generators, and Case of Water Pumps are 0, 268, 166, and 16 respectively. That in turn, enable company to make 145232.9 USD profit. In addition to cost, these numbers enable company to fully utilize warehouse area. So, in that case, Cost and Warehouse area are binding constraints.

Additionally, I utilized R in order to see what will happen if increase total available cost while keeping others constant. From Figure 1.1, we can see that it is wise to increase total allowed money till 350000 USD. By doing so, company can increase their profit up to almost 210000 USD. After that, since warehouse are is limited , it does not give any benefit to increase allowed monthly cost.

*Figure 1.1 – Profit values vs cost amount (in 1000’s)*

# Sensitivity Analysis

One of the interesting things from first part is that, optimal value for Pressure washer is 0. It does make sense since both Generators and Pressure washers occupy same area in the inventory whereas Generator produce 140 USD more profit. Since there is not another constraint regarding only these two, it is not wise to Sell Pressure washers when they are inferior to Generators. So, I utilized Excel Solver in order to generate sensitivity analysis for decision variables.

*Table 1.3 – Sensitivity Report for decision variables*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Name | Value | Cost | Coefficient | Increase | Decrease |
| Pressure Washer | 0 | -103,0 | 119,99 | 103,04 | 1E+30 |
| Go Kart | 268 | 0,0 | 359,99 | 362,66 | 137,172787 |
| Generator | 166 | 0,0 | 224,99 | 182,40 | 148,6891 |
| Case of 5 Water Pumps | 17 | 0,0 | 712,95 | 1824,02 | 497,3296199 |

From Table 1.3 , we can see that , we should increase the price of Pressure Washer at least 103.4 USD in order to sell nonzero amount of them.

# Conclusion

To conclude, in this analysis, my main aim was to utilize linear programming in order to perform optimization analysis for a specific company. I was given a specific business scenario and asked to design linear model in order to maximize company’s profit bounded with 4 constraints. Constraints were about cost, area and company’s specific marketing policy. After my analysis, I found that optimal numbers for Pressure Washer, Go Karts, Generators and case of Water Pumps were 0, 268, 166, and 16 respectively. This, in turn, enabled company to make 145232.9 USD profit. On the other hand, I found that company can think of increasing the allowed monthly cost up to almost 350000 USD which can increase their profit up to almost 210000 USD, keeping all thing equal. After that point , due to other constraints, it is not wise to increase monthly budget. Moreover, in this condition, where total cost is bounded by 176000, I observed that their inventory is fully utilized and there is not excess space which create unnecessary cost. Only thing company can improve is to reconsider length of shelves and in order to utilize them fully. Instead of 32 ft shelves, they can use24 ft shelves which is optimal for both 8 ft and 3 ft products.

For the second part, I dived deeper in order to find the optimal price where number of Pressure Washers will be nonzero number. I found that we should increase it price by 103.4 USD in order to get nonzero amount.

# References

Linear programming in R. (2018, August 16). Retrieved from

<https://www.r-bloggers.com/linear-programming-in-r/>